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quoted. But in addition to this we see that the definition assumes that the circle divides its plane into at least two parts, one of which is *within*. We are then supplied with other information, such as, that *since* a circle is a *closed curve*, a line from any point *within* to any point *without* the circle *intersects* the circle at least once. This assumes, among other things, that the locus is continuous; yet how all these conclusions are to be deduced from the definition, it would be difficult to state.

Most of the preliminary data in the case of circles, as given in current texts, is based entirely on our visual perception of the figure, with no attempt at deducing it from the definition; none of their off-hand statements are more evident at a glance than the fact that a straight line cannot ''intersect'' a circle in more than two points, yet a capricious sense of logical rigor requires that this assertion be demonstrated at length.

The premature definition of secants, tangents, tangent circles, etc., in current texts, tends to dull the sense of logical order.

We are not surprised, when the definition given of a circle is similar to one of the two quoted, to find many fallacies in their treatment of these figures. For example, consider the following: 'The line joining a point to the center of a circle is less than, or greater than, a radius, according as the point is within, or without, the circle. Let P be a point within, and Q a point without, a circle whose center is O. *Extend* OP to *meet* the circle in A, and *let* OQ *cut* the circle in C.' These statements are the very facts we wish to prove; namely, that C lies between O and Q, and that A does not lie between O and P.

Generally, with no consideration of the relative magnitudes of arcs of a circle, we are told that a chord subtends two arcs, of which the *lesser* is meant unless otherwise stated. Most of the subject as usually treated is equally vulnerable.

ON KINEMATIC GEOMETRY.—A NEW INVERSOR.

By JOHN J. QUINN, Warren, Pa.

The Theory of Inversion, an important method of investigation especially in many problems of mathematical physics, is one of the distinctive features of modern geometry. It is a method of transformation whereby one surface, or curve, is represented by another surface or curve in a manner that insures similarity in the most minute details. In general, the inverse of a sphere is a sphere; and the inverse of a circle is a circle. In order that two points be in inverse relation it is necessary that they fulfil two conditions:

1. *They must be always collinear with a fixed point.*
2. *The product of their distances from the fixed point must be constant.*

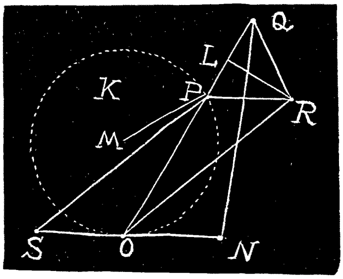
By harnessing this principle it is possible to draw a line that is mathematically straight. This can be done by a linkwork so constructed that when the whole system is moved there will be three points, as O, P, and Q collinear in every position, and in the relation $OP \times OQ$=constant.

A linkwork that will describe the inverse of a given curve or surface is termed an Inversor.

The inverse points as O, P, and Q, in an inversion are called the foci.

THEOREM. *Let the side OS of the parallelogram ORPS be extended to N, making ON equal to OS, and located so that NQ equals OR, and RQ equals PR. Then if this figure be embodied in a linkage:*

1. *The points O, P, and Q, are always collinear.*
2. *The points O, P, and Q, are inverse points.*
3. *If the focus O of the linkage be pivoted to a fixed base, and the focus P be constrained to move in a circle through O, the focus Q will describe a straight line.*



Given: $SP=OR=NQ$; $SO=ON=PR=RQ$.

Proof 1. $\triangle ONR \cong \triangle QNR$.

But $OP \parallel NR$.

∴The points O, P, and Q are collinear.

Proof 2. Draw RL perpendicular to OPQ. Then $OP=OL-LP=\sqrt{(OR^2-RL^2)}-\sqrt{(PR^2-RL^2)}$; $OQ=OL+LP=\sqrt{(OR^2-RL^2)}+\sqrt{(PR^2-RL^2)}$.

∴ $OP.OQ=OR^2-PR^2$=a constant.

∴The points O, P, and Q are inverse points.

Proof 3. ∵ O, P, and Q are inverse points, and since a straight line is the inverse of a circle through the pole of inversion,

∴The line QV is a straight line, the inverse of the circle K. Q. E. D.

SIX PROPOSITIONS ON PRIME NUMBERS.

By R. D. CARMICHAEL, Hartselle, Ala.

PROPOSITION A. *If p is an odd prime,* $(1.2.3.....\frac{p-1}{2})^2+(-1)^{\frac{1}{2}(p-1)}\equiv 0$ $(mod\ p)$.

By Wilson's theorem, if p is prime

$$1.2.3.....(p-1)+1\equiv 0 \pmod p \qquad (1).$$

This may be written

$$(1.2.3.....\frac{p-3}{2}.\frac{p-1}{2})(p-\frac{p-1}{2})(p-\frac{p-3}{2}).....(p-2)(p-1)+1\equiv 0 \pmod p \qquad (2).$$